

Oscillations of Cumulant Moments - Universality of Amplitudes

M.Rybczyński¹, G.Wilk^{2*}, Z.Włodarczyk^{1†},
M.Biyajima^{3‡} and N.Suzuki^{4§}

¹*Institute of Physics, Pedagogical University,
Konopnickiej 15; 25-405 Kielce, Poland*

²*The Andrzej Soltan Institute for Nuclear Studies,
Hoża 69; 00-689 Warsaw, Poland*

³*Department of Physics, Shinshu University,
Matsumoto 390, Japan*

⁴*Matsusho Gakuen Junior College,
Matsumoto 390-12, Japan*

February 1, 2008

*e-mail:wilk@fuw.edu.pl

†e-mail:wlod@pu.kielce.pl

‡e-mail:biyajima@azusa.shinshu-u.ac.jp

§e-mail:suzuki@matsu.ac.jp

Abstract

We demonstrate on simple examples that oscillatory behaviour of moments of multiplicity distributions $P(n)$ observed in e^+e^- annihilations, in hadronic pp collisions and in collisions on nuclei, $p + A$, is to a large extent caused by the experimental artifact of measuring only limited range of $P(n)$. In particular we show that by applying a suitable universal cut-off procedure to the measured $P(n)$ one gets for reactions mentioned before oscillations of similar magnitude. The location of zeros of oscillations as a function of the rank of moments and their shapes remain, however, distinctively different for different types of reactions considered. This applies to some extent also to collisions of nuclei, which otherwise follow their own pattern of behaviour.

PACS numbers: 13.65.+i 13.65.-7 24.60.-k

The problem of the possible physical origin and information content of oscillations in the cumulant moments of the corresponding multiplicity distributions $P(n)$ started with QCD calculation of the respective generating functional. It turned out that the resulting cumulant moments oscillate as a function of their rank in the way depending on the QCD parameters used [1]. This finding was confirmed by the analysis of e^+e^- and hadronic pp data, which showed that, indeed, the q -th rank normalized cumulant moment of observed negatively charged multiplicity distributions oscillates irregularly around zero with increasing q (the minimum points being located around $q \simeq 5$). There was therefore a hope that analysis of these oscillations can then prove a crucial test for the QCD [2].

These expectations were, however, soon confronted with observations that the same data can be equally well described by a more phenomenological methods [3, 4] based on the solutions of stochastic processes [5], for example by the negative binomial distribution (NBD) [6] (in its truncated version) or by the modified negative binomial distribution (MNBD) [6, 7]. The interesting finding was that the NBD and MNBD differ distinctively in this context in the following sense: for untruncated multiplicity distributions the q -th rank normalized cumulant moment of the NBD is always positive and decreases monotonically with increasing q , whereas for MNBD it can oscillate in the way depending on the choice of parameters. Therefore, in this approach, the behavior of cumulant moments obtained from experimental data seemed to provide a new constraint on models of multiplicity distributions. In particular it was shown in [3, 4] that cumulant moments of negatively charged particles in e^+e^- collisions can be described by both the truncated NBD and (truncated or not) MNBD (which performs better in the e^+e^- case).

The observation that truncation of the NBD makes the corresponding moments oscillate has confirmed the statement made before in Ref. [8]. It was said there that important (if not exclusive) factor leading to oscillations of moments is the experimental fact of necessary truncation of the observed $P(n)$ at some maximal multiplicity n_{max} . The observed differences between results from different reactions seem to reflect therefore only the level of this truncation.

Cumulant moments obtained from the hA experimental data also oscillate with magnitude which is much bigger than that observed in hh collisions [9]. The specific feature of these oscillation is that they can be attributed not only to truncation of $P(n)$ but also to the fact that the number of elementary collisions in the hA reac-

tion is necessary limited by the geometry of collision [10]. This is, in fact, a kind of truncation as well, but this time it is caused by the geometry of the collision rather than by the experimental setup. This new geometrical factor should be therefore even more important in heavy ion collisions [11, 12].

The above discussion clearly shows that information content of the oscillation phenomenon remains still unclear and subject to debate. The aim of our note is thus to shed a new light on this problem by discussing a couple of simple but illustrative numerical examples of oscillations in e^+e^- annihilations, hadronic pp collisions and collisions involving nuclei, pA . We shall not attempt here a fit to experimental data because this was done already in the relevant works quoted here. Our intention was rather to use the existing experience on this subject (especially that contained in [3, 4, 10]) in order to demonstrate a possible universality existing in the e^+e^- , pp and pA data on oscillations of moments. We shall also address, albeit only shortly, heavy ion collisions, AB , in this context.

We shall use, as is usually done, the following moments of the multiplicity distribution $P(n)$ (cf. [10]):

$$H_q = \frac{K_q}{F_q} \quad (1)$$

where

$$K_q = \frac{k_q}{\langle n \rangle^q}, \quad F_q = \frac{f_q}{\langle n \rangle^q} \quad (2)$$

with k_q and f_q being the usual cumulant and factorial moments of rank q of $P(n)$. What is observed experimentally is the fact that they oscillate and that these oscillations differ substantially depending on the type of reaction and do it in two ways:

- their amplitudes vary, increasing from the value of 10^{-3} for e^+e^- annihilations, via the value of 10^{-2} for pp collisions up to the value of 10^{-1} for pA and heavy ion (AB) reactions;
- their shapes are different with frequency of oscillations in q being highest for e^+e^- reactions.

As was said before, the main cause of these oscillation is supposed to be experimental truncation of the corresponding $P(n)$. In order to compare results of such truncation for different reactions we propose to use a universal variable u defined in the following way:

$$u = \frac{n_{max} - \langle n \rangle}{\sigma_n}. \quad (3)$$

This variable measures distance of the cut-off point, n_{max} , from the (specific for the process under consideration) mean multiplicity $\langle n \rangle$. It does this in terms of standard deviation σ_n (which is obtained from the same $P(n)$). In this way it allows to compare results from different reactions by providing a kind of natural and universal measure for terminating $P(n)$ under considerations at some value of n_{max} .

In Fig. 1 we show examples of H_q moments calculated for e^+e^- , pp and pA reactions for two different choices of the values of cut-off parameter u . In each case we have used identical multiplicity distributions (and all other relevant parameters) as those used in Refs. [3, 4, 10] when describing the same reactions. They were then cut-off for each reaction considered at the same value of the variable u defined in eq. (3) and from them the corresponding moments H_q were calculated. As can be seen, cutting off multiplicity distributions $P(n)$ (calculated for different reactions) at the same values of u results in comparable values of amplitudes of observed oscillations. Although they are still not identical, the previously mentioned differences in amplitudes are enormously reduced, being now of the same order of magnitude. This feature apparently does not depend on the actual value of variable u used (although the changes of u affect the shape of oscillations). It proofs therefore that the increase of amplitudes of oscillations observed between e^+e^- and pA reactions is caused mainly by different experimental cut-off procedures (quantified here by different values of variable u used in the respective processes) applied to the measured $P(n)$. In e^+e^- processes, with smallest amplitudes of oscillations, the $P(n)$ were measured most accurately, up to the very high multiplicities (i.e., to large values of the ratio $z = n/\langle n \rangle$). The opposite situation is encountered in pA processes. This is the main result of our note.

This kind of universality (even if only approximate) makes the sizes of amplitudes of oscillations not particularly sensitive to the dynamical details of $P(n)$ of interest. Not much is left in this observable when different experiments, but with the same values of variable u , are compared with each other. On the other hand, the character of oscillations, as visualised by their frequency in the rank q of moments, remains in a visible way different for different types of reactions and can therefore be used for dynamical discrimination between different models. For example, in Fig. 2 we show oscillations of H_q moments obtained for the same value of $u = 7$ using $P(n)$ in the form of MNBD as given in [3, 4]:

$$P(0) = \frac{(1 + r_1)^N}{(1 + r_2)^{N+k}},$$

$$P(n) = \frac{1}{n!} \left(\frac{r_1}{r_2}\right) \sum_{j=0}^N {}_N C_j \frac{\Gamma(k+n+j)}{\Gamma(k+j)} \left(\frac{r_2-r_1}{r_1}\right)^j \frac{r_2^n}{(1+r_2)^{n+k+j}}, \quad (4)$$

where N , k , $r_{1,2}$ are parameters. Referring to [3, 4] for details we shall say only that if $k = 0$, the summation in eq. (4) runs from $j = 1$ up to $j = N$ and the resultant distribution is called MNBD. In this case parameters $r_{1,2}$ are given by the average multiplicity $\langle n \rangle$ and second moment C_2 of the corresponding $P(n)$ as $r_{1,2} = \frac{1}{2} \left(C_2 - 1 - \frac{1}{\langle n \rangle} \mp \frac{1}{N} \right) \langle n \rangle$. For $N = 0$, parameter r_1 disappears from (4) and it reduces to the NBD with $r_2 = \frac{\langle n \rangle}{k}$. The parameters r_1 and r_2 of the NMBD reflect now the structure of oscillations rather than their amplitudes. As one can see they change systematically from parameters describing e^+e^- annihilation (left-top panel) to those typical for hadronic pp collisions (right-bottom panel) [3, 4].

To summarize, we stress again that the magnitude of observed oscillations of H_q moments of multiplicity distributions $P(n)$ reflect essentially our ability to measure, in a given reaction, large multiplicities. When analysing data using the same value of our universal cut-off parameter u one gets comparable values of amplitudes for all reactions of interest. It means that this quantity is not sensitive to dynamical details of reaction. The shape of oscillations remains, however, sensitive to such details. It can therefore be used to extract a new dynamical information from different multiplicity distributions (when compared at the same values of the cut-off parameter u).

The separate issue is the problem of oscillations in heavy ion collisions AB , which we should now briefly address for the sake of the completeness of presentation. They do not share the property discussed above. The reason is the following. As was already mentioned, in the collisions of two nuclei, A and B , the nuclear geometry is the main factor responsible for the shape and properties of the corresponding multiparticle distribution of produced secondaries $P(n)$ [11]. This fact is crucial in generating oscillations in the respective cumulant moments. To show it on some example let us first write the typical corresponding multiplicity distribution for $A+B$ collision:

$$P(n) = \sum_{\mu=1}^{\mu_{tot}} p(\mu) \prod_{i=1}^{\mu} P_i(n_i) \delta \left(n - \sum_{i=1}^{\mu} n_i \right). \quad (5)$$

It contains two ingredients: distribution $p(\mu)$ of the number of emitting sources μ and the respective “elementary” multiplicity distributions of particles produced from such sources, $P_i(n_i)$. The emitting sources can be, for example, understood as in [13]. Their distribution can be calculated in the same way as in [10, 11, 12]. In the example below we have used a simple Monte Carlo code in which two colliding nuclei consisting of A and B nucleons, respectively, collide with each other.

Nucleons are distributed in nucleus according to a standard Saxon-Woods (SW) distribution (with diffusiveness 0.49 fm for S and 0.545 fm for Pb nuclei and corresponding nuclear radii given by the formula: $r[\text{fm}] = 1.12A^{1/3} - 0.86/A^{1/3}$). They collide with each other with probability given by their (total inelastic) cross section $\sigma = 32$ mb. This provides us with $p(\mu)$. On the other hand $P_i(n_i)$ has been taken again from the MNBD fits to elementary collisions performed in [3, 4]). In nuclear collisions two distinct classes of events occur and must be treated separately: central and minimum bias collisions. In our case central collisions were chosen as 1% of the collisions with smallest impact parameter. In Fig. 3 we show results for moments H_q of $P(n)$ from eq. (5) calculated for $S + S$ (left panels) and $Pb + Pb$ (right panels) minimum bias (upper panels) and central (lower panels) collision for the same values of the variable $u = 5$. Notice that the magnitude of amplitude of oscillations (especially for central collisions) remains different from that in the corresponding panels of Fig. 1. It means that in this case there is no such universality as in the previously discussed reactions. On the other hand, however, minimum bias collisions are distinctively different from the central ones, which show only very small oscillations. The patterns shown apparently depend only weakly on the choice of the colliding nuclei (i.e., on the parameters of the Monte Carlo producing $p(\mu)$).

To understand better results presented in Fig. 3 one should realise that central collisions result in a large number of elementary collisions, i.e., in a large number of emitting sources μ in each event. Therefore, because of central limit theorem, irrespectively of details of elementary collisions $P(n)$ must have a gaussian-like shape. We can parametrize it as: $P(n) = P_0 \cdot \exp\left(-\frac{(n-n_0)^2}{2a}\right)$. On the other hand, the minimum bias collisions result in $p(\mu)$ of the box-like, or Saxon-Woods (SW)-like shape and such will be also resultant $P(n)$: $P(n) = \frac{P_0}{1 + e^{(n-n_0)/a}}$. In Fig. 4 we show, as illustration, some typical examples of oscillation patterns emerging from both types of distributions. Notice that whereas we essentially observe no oscillations in the case of gaussian $P(n)$ (or, if at all, they do start at large q), we see strong oscillations for the SW $P(n)$. They are caused in this case by the box-like shape of the SW distribution, which is best demonstrated by the fact that they gradually vanish with the increasing diffusiveness of SW used, i.e., with the increasing values of parameter a [14]. It should be pointed here that results presented in Fig. 4 were obtained without additional truncation in multiplicity, i.e., in (3) $n_{tot} = \infty$. All oscillations present there are thus entirely of different origin than the simple truncation of $P(n)$. They are governed by the geometrical parameter a and by the level of observability of the total $P(n)$.

Summarizing, we have demonstrated (approximate) universality of amplitudes of oscillations of cumulant moments when compared at the same values of the variable u as defined in (3). It shows up for a range of reactions from e^+e^- annihilation processes, via hadronic pp collisions, to pA reactions. The latter start to show influence of the geometry of collision process, which entirely dominates the truly nuclear collisions.

Acknowledgements

G.W. would like to extend his gratitude to the Physic Department of Shinshu University and to Matsusho Gakuen Junior College for their warm hospitality during his visit to Matsumoto where this work originated. M.B. is partially supported by the Grant-in Aid for Scientific Research from the Ministry of Education, Science and Culture (No. 06640383) and by the Exchange Program between JSPS and the Polish Academy of Science. N.S. thanks for the financial support by Matsusho Gakuen Junior College.

References

- [1] I. M. Dremin and V. A. Nechitailo, *JETP Lett.* **58** (1993) 881;
I. M. Dremin and R. Hwa, *Phys. Rev.* **D49** (1994) 5805.
- [2] I. M. Dremin, et al. *Phys. Lett.* **B336** (1994) 119.
- [3] N. Suzuki, M. Biyajima and N. Nakajima, *Phys. Rev.* **D53** (1996) 3582 and **D54** (1996) 3653.
- [4] N. Nakajima, M. Biyajima and N. Suzuki, *Phys. Rev.* **D54** (1996) 4333.
- [5] M. Biyajima and N. Suzuki, *Phys. Lett.* **143B** (1984) 463 and *Prog. Theor. Phys.* **73** (1985) 918.
- [6] N. Suzuki, M. Biyajima and G. Wilk, *Phys. Lett.* **B268** (1991) 447.
- [7] P. V. Chliapnikov and O. G. Tchikilev, *Phys. Lett.* **B242** (1990) 275 and **B282** (1992) 471;
P. V. Chliapnikov, O. G. Tchikilev and V. A. Uvarov, *Phys. Lett.* **B352** (1995) 461.

- [8] R. Ugoccioni, A. Giovannini and S. Lupia, *Phys. Lett.* **B342** (1995) 387.
- [9] A.Capella, I.M.Dremin, V.A.Nechitailo and J.Tran Than Van, *Z.Phys.* **C75** (1997) 89;
I. M. Dremin, V. A. Nechitailo, M. Biyajima and N. Suzuki, *Phys. Lett.* **B403** (1997) 149.
- [10] N.Suzuki, M.Biyajima, G.Wilk and Z.Włodarczyk, *Phys. Rev.* **C58** (1998) 1720.
- [11] W.Q.Chao and B.Liu, *Z. Phys.* **C42** (1989) 337; P.Zhuang and L.Liu, *Phys. Rev.* **D42** (1990) 848.
- [12] J.Dias de Deus, C.Pajares and C.A.Salgado, *Phys. Lett.* **B407** (1997) 335.
- [13] M. Biyajima, N. Suzuki, G. Wilk and Z. Włodarczyk, *Phys. Lett.* **B386** (1996) 297.
- [14] It is interesting to mention that making $p(\mu)$ more diffuse (similar to $a = 99$ case in Fig. 4) results in the vanishing of the oscillation pattern (in spite of the fact that MNBD used for elementary $P_i(n_i)$ lead always to oscillations of moments in elementary collisions [3, 4]).

Figure captions

- Fig. 1** Examples of H_q moments of multiplicity distributions for the e^+e^- annihilation (upper panels), pp reactions (middle pannels) and pA reactions (bottom panels) for two chosen values of the parameter u : $u = 5$ (left panels) and $u = 8$ (right pannels). (The $P(n)$ data are the same as in [1, 2, 3, 4, 10]).
- Fig. 2** The H_q moments obtained from the MNBD for $P(n)$ for different values of its characteristic parameters r_1 and r_2 (cf. [3, 4]). The upper-left panel corresponds to e^+e^- and bottom-right one to pp reactions, respectively. The value of parameter $u = 7$ reamains all time the same. Notice the gradual change of frequency of oscillations whereas their amplitudes remain essentially of the same order of magnitude.
- Fig. 3** The H_q moments calculated for $S + S$ (left panels) and $Pb + Pb$ (right panels) collisions of the minimum bias (upper panels) and central (lower panels) type. In both cases $u = 5$.
- Fig. 4** Examples of oscillation patterns for gaussian-like (upper panels) and SW-like (lower panels) shapes of multiplicity distributions $P(n)$, see text for details. Short dash, long dash and full lines correspond to parameter a equal to $a = 2, 20, 80$ for gaussian-like distributions and to $a = 99, 50, 10, 0.001$ for SW-like distributions; in both cases $n_0 = 400$.

Figure 1

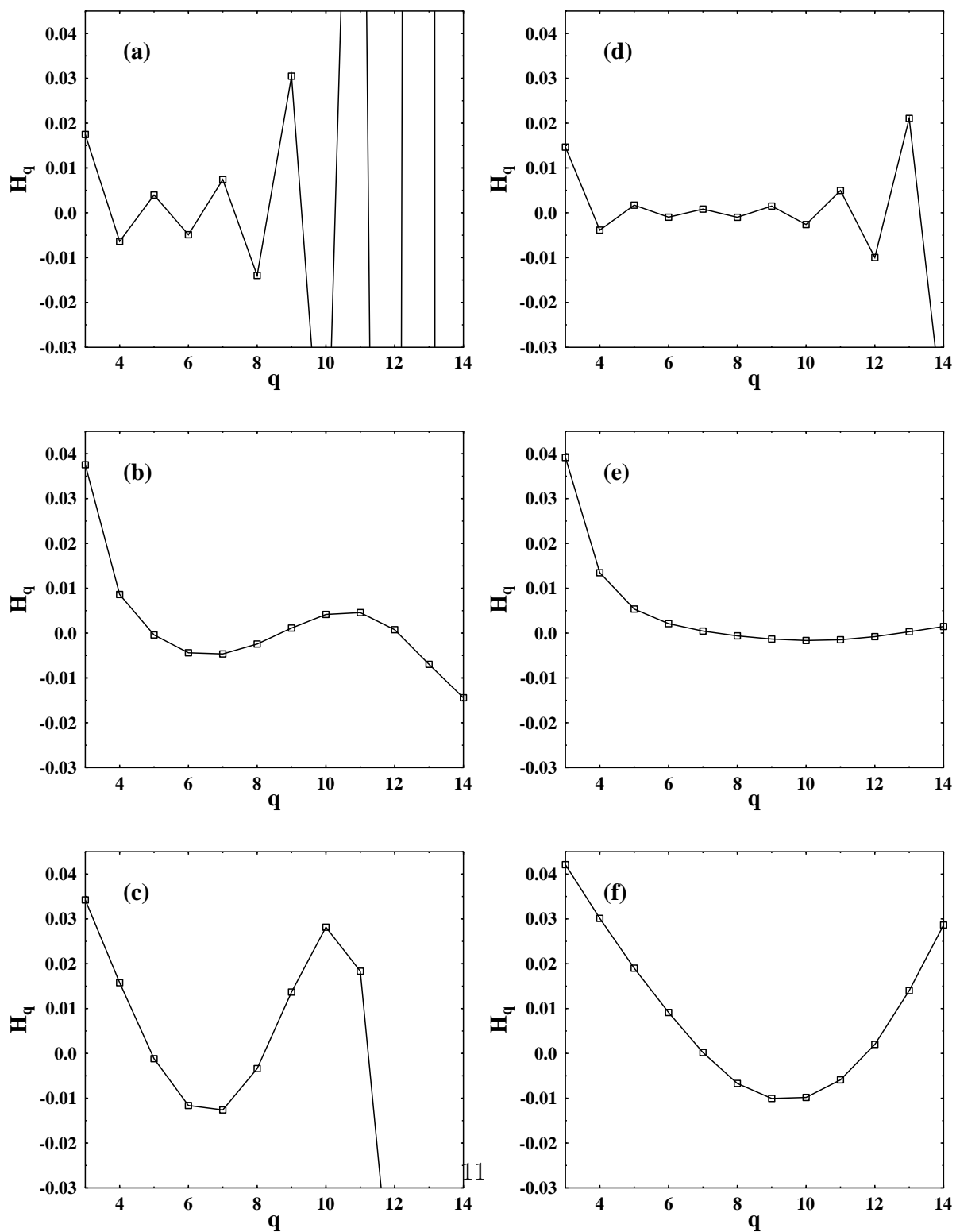


Figure 2

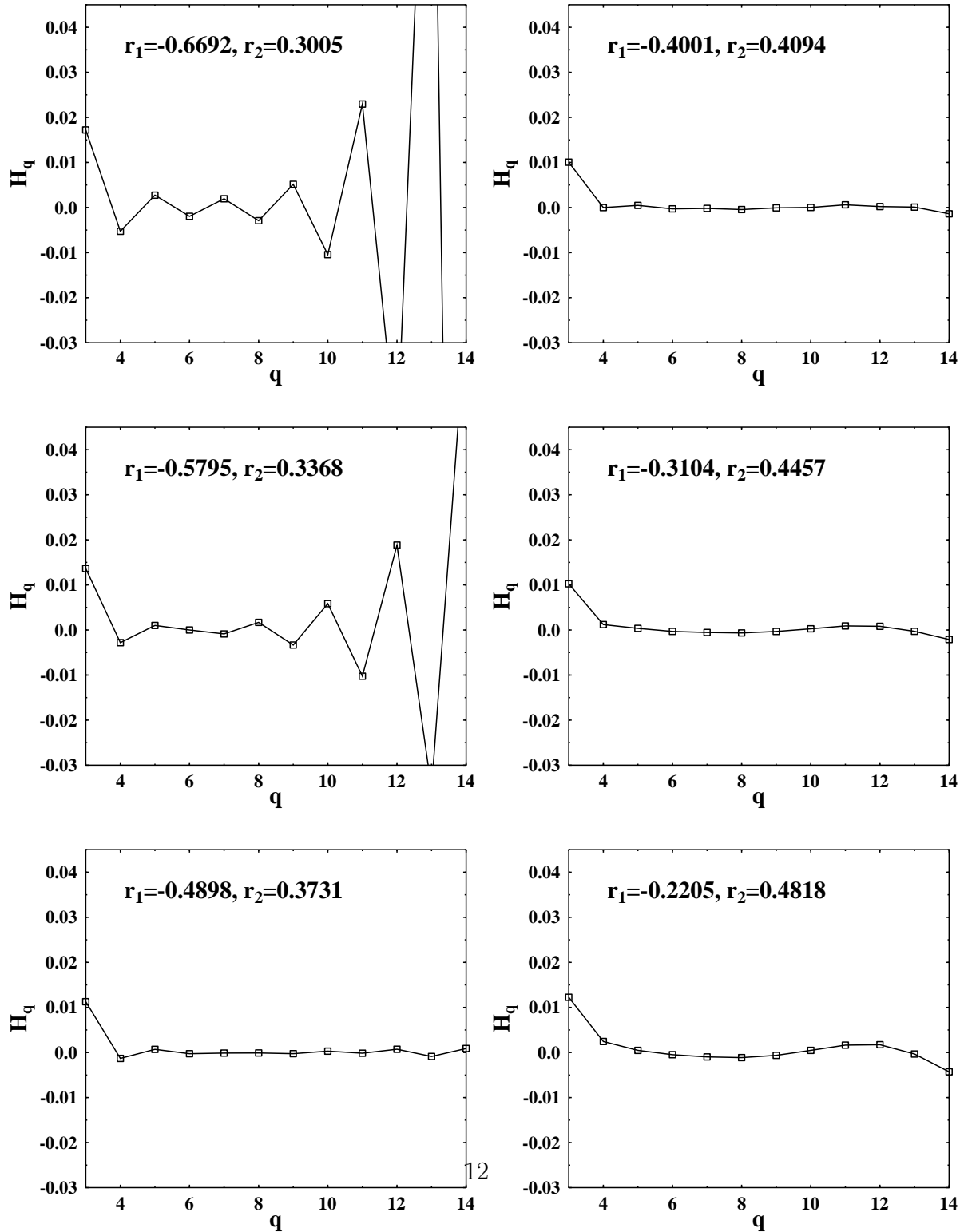


Fig. 3

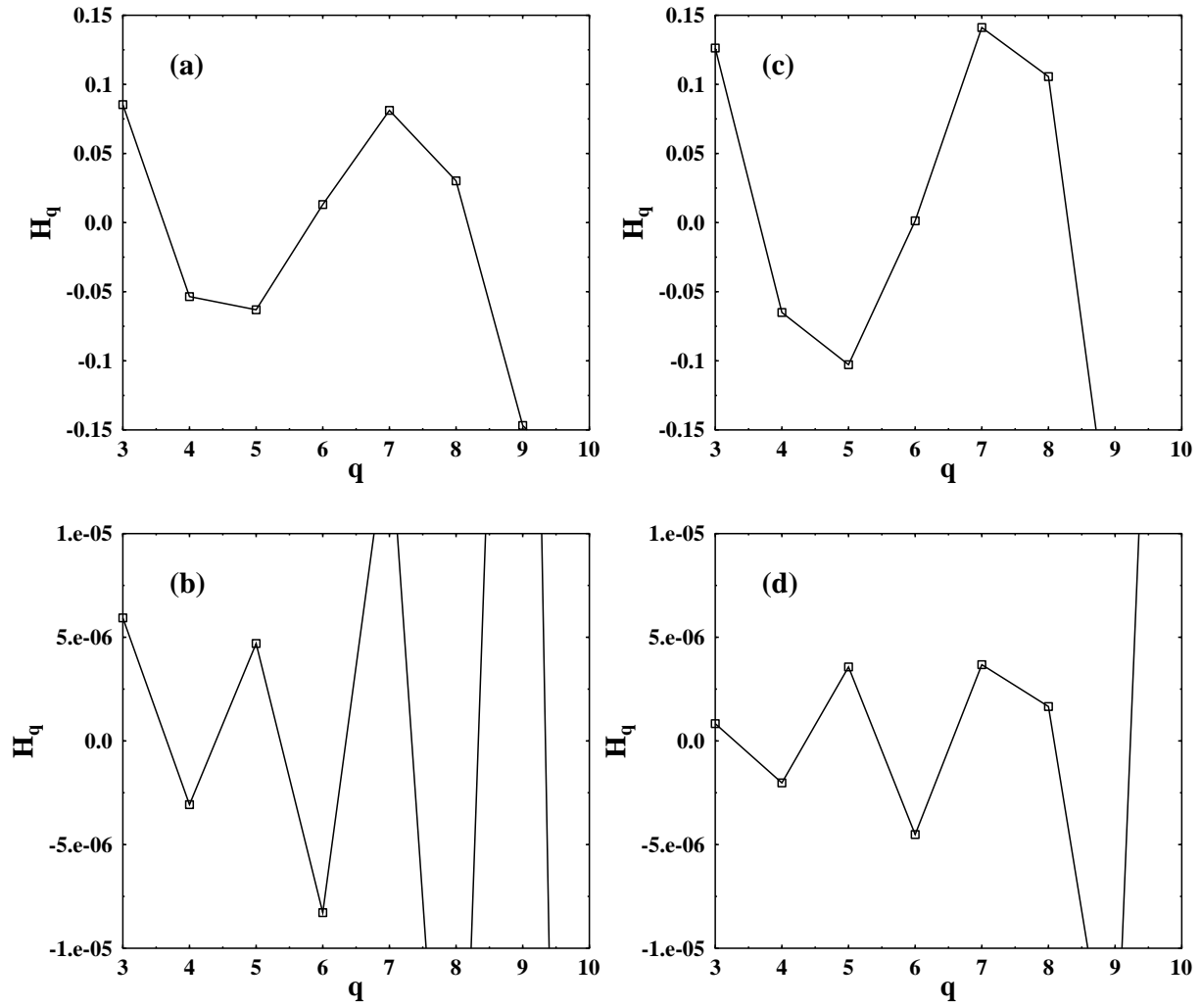


Fig. 4

